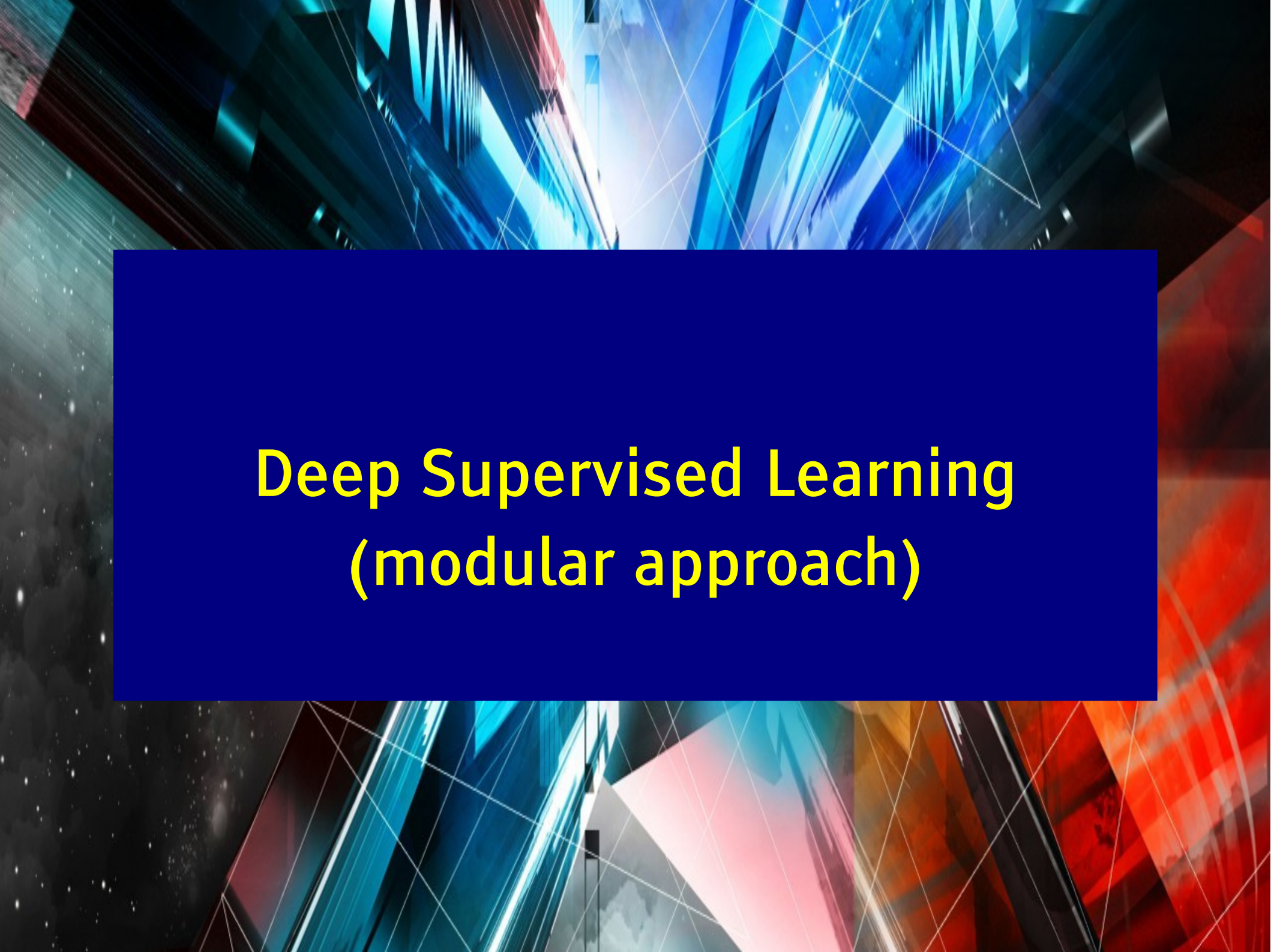
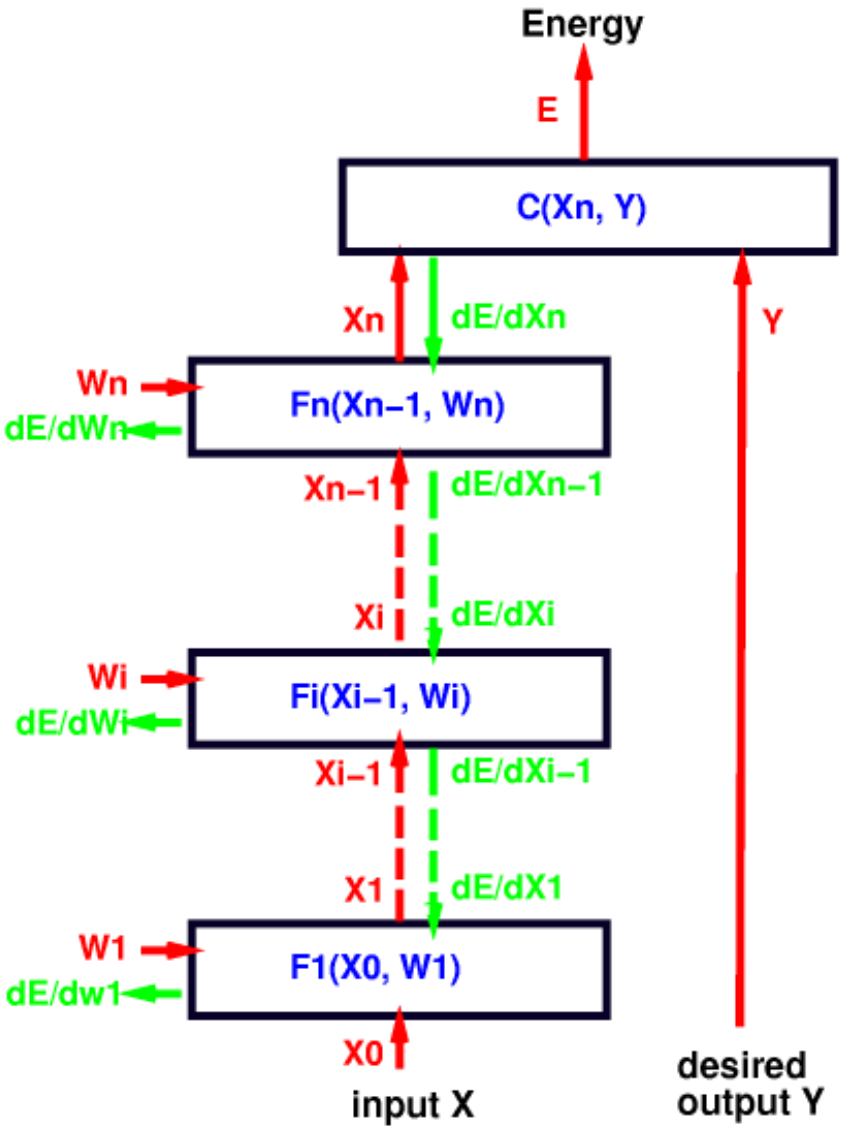

Deep Supervised Learning
(modular approach)

Multimodule Systems: Cascade

Y LeCun
MA Ranzato



- Complex learning machines can be built by assembling modules into networks
- Simple example: sequential/layered feed-forward architecture (cascade)
- Forward Propagation:

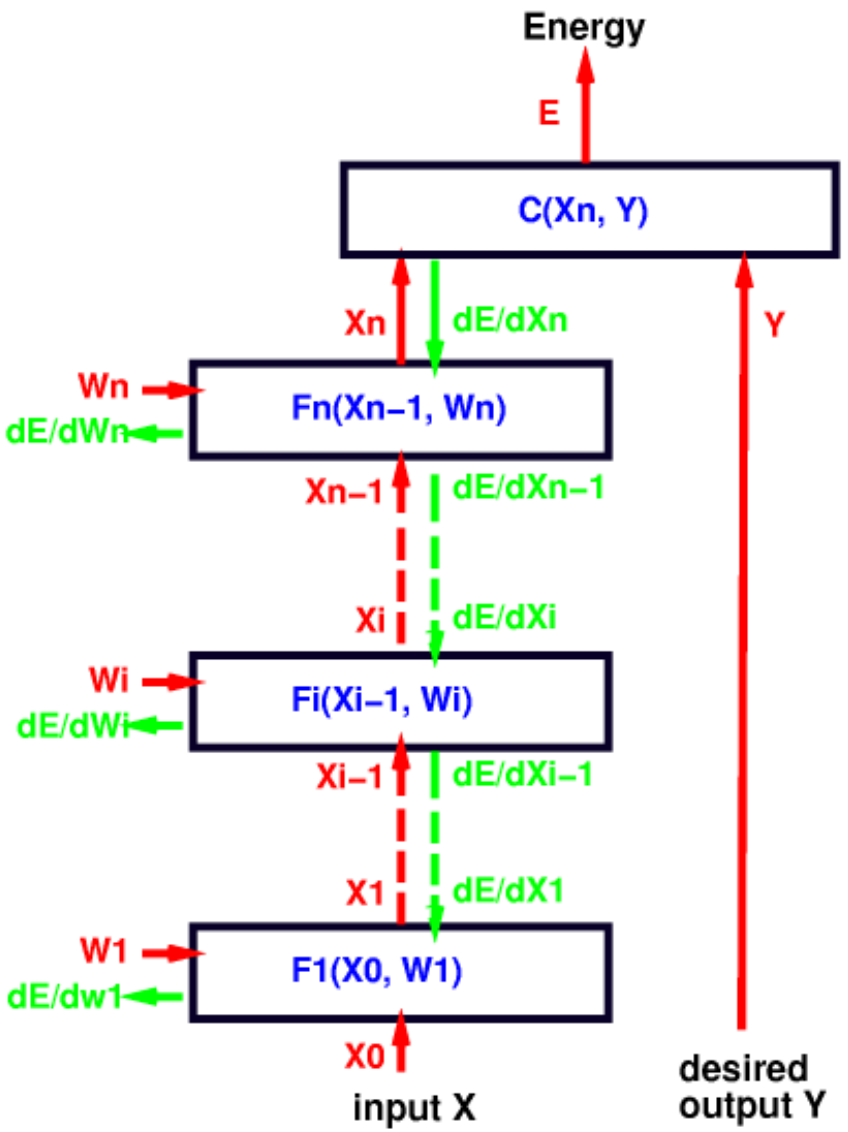
$$\text{let } X = X_0,$$

$$X_i = F_i(X_{i-1}, W_i) \quad \forall i \in [1, n]$$

$$E(Y, X, W) = C(X_n, Y)$$

Multimodule Systems: Implementation

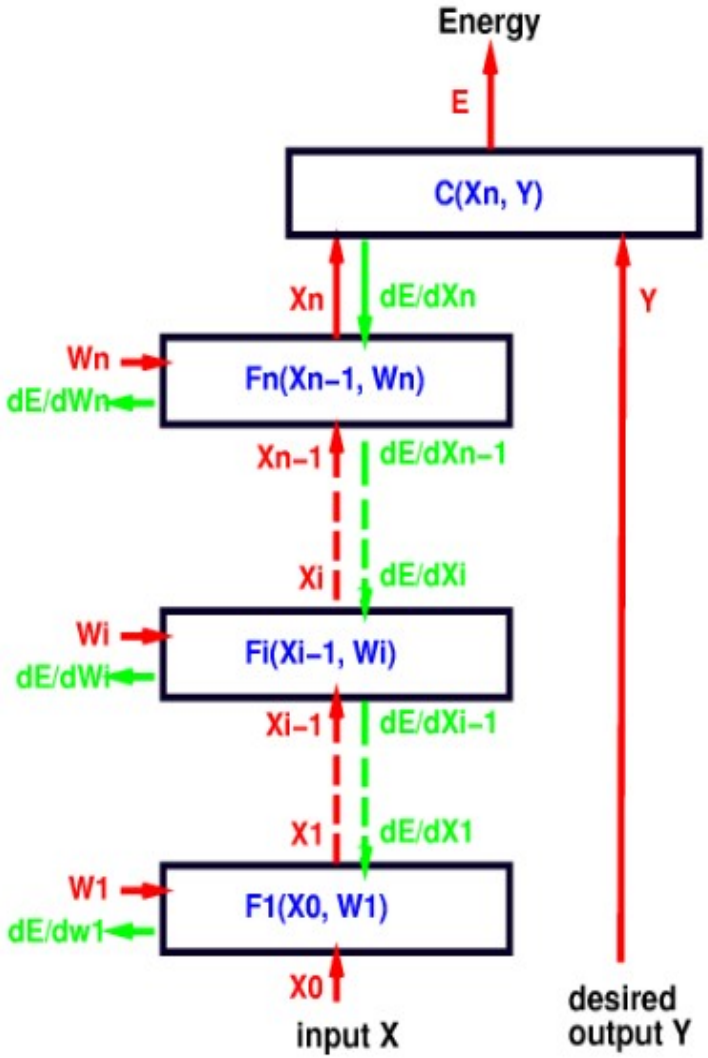
Y LeCun
MA Ranzato



- Each module is an object
  - Contains trainable parameters
  - Inputs are arguments
  - Output is returned, but also stored internally
  - Example: 2 modules m1, m2
- Torch7 (by hand)
  - `hid = m1:forward(in)`
  - `out = m2:forward(hid)`
- Torch7 (using the nn.Sequential class)
  - `model = nn.Sequential()`
  - `model:add(m1)`
  - `model:add(m2)`
  - `out = model:forward(in)`

Computing the Gradient in Multi-Layer Systems

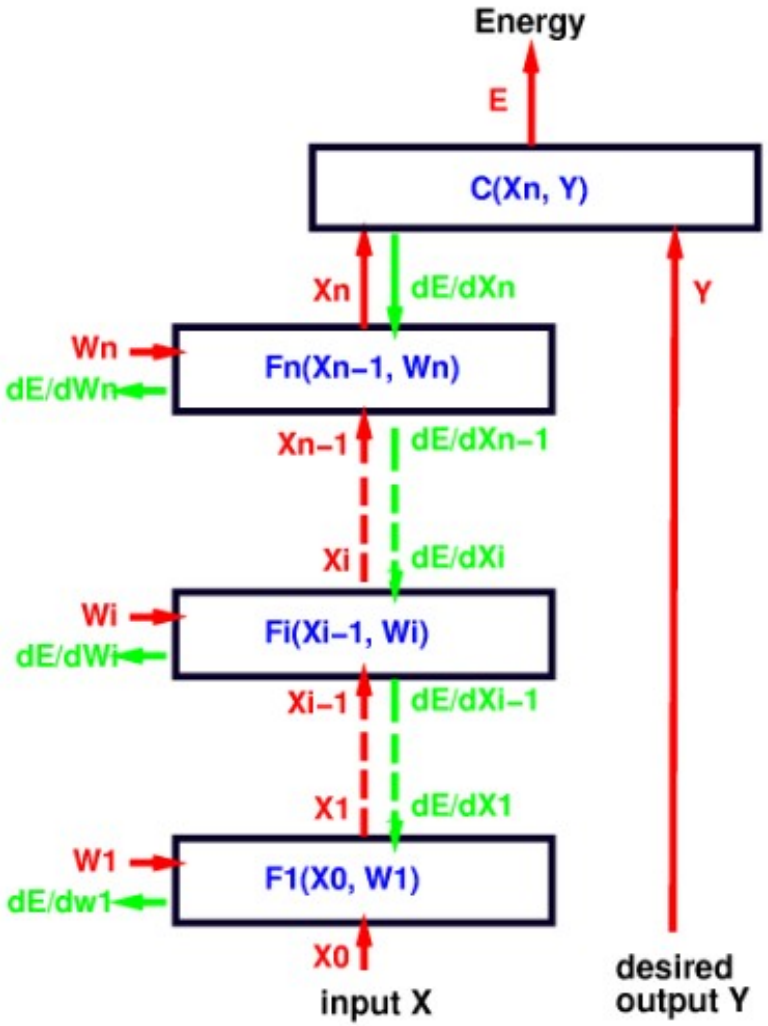
Y LeCun
MA Ranzato



- To train a multi-module system, we must compute the gradient of E with respect to all the parameters in the system (all the W_i).
- Let's consider module i whose fprop method computes $X_i = F_i(X_{i-1}, W_i)$.
- Let's assume that we already know $\frac{\partial E}{\partial X_i}$, in other words, for each component of vector X_i we know how much E would wiggle if we wiggled that component of X_i.

Computing the Gradient in Multi-Layer Systems

Y LeCun
MA Ranzato



- We can apply chain rule to compute $\frac{\partial E}{\partial W_i}$ (how much E would wiggle if we wiggled each component of W_i):

$$\frac{\partial E}{\partial W_i} = \frac{\partial E}{\partial X_i} \frac{\partial F_i(X_{i-1}, W_i)}{\partial W_i}$$

$$[1 \times N_w] = [1 \times N_x].[N_x \times N_w]$$

- $\frac{\partial F_i(X_{i-1}, W_i)}{\partial W_i}$ is the *Jacobian matrix* of F_i with respect to W_i.

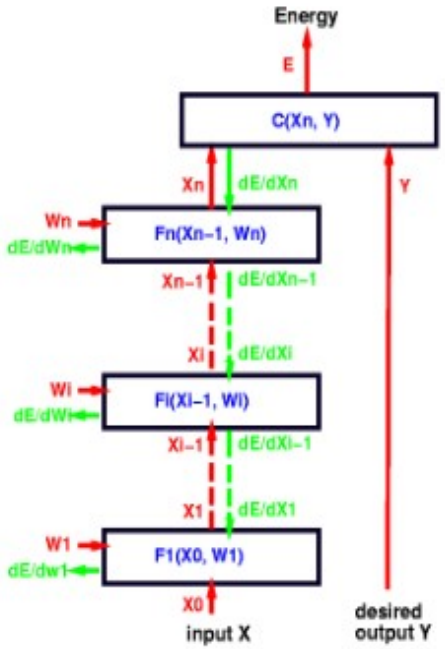
$$\left[\frac{\partial F_i(X_{i-1}, W_i)}{\partial W_i}\right]_{kl} = \frac{\partial \left[F_i(X_{i-1}, W_i)\right]_k}{\partial [W_i]_l}$$

- Element (k, l) of the Jacobian indicates how much the k-th output wiggles when we wiggle the l-th weight.

Computing the Gradient in Multi-Layer Systems

Y LeCun
MA Ranzato

Using the same trick, we can compute $\frac{\partial E}{\partial X_{i-1}}$. Let's assume again that we already know $\frac{\partial E}{\partial X_i}$, in other words, for each component of vector X_i we know how much E would wiggle if we wiggled that component of X_i.



- We can apply chain rule to compute $\frac{\partial E}{\partial X_{i-1}}$ (how much E would wiggle if we wiggled each component of X_{i-1}):

$$\frac{\partial E}{\partial X_{i-1}} = \frac{\partial E}{\partial X_i} \frac{\partial F_i(X_{i-1}, W_i)}{\partial X_{i-1}}$$

- $\frac{\partial F_i(X_{i-1}, W_i)}{\partial X_{i-1}}$ is the *Jacobian matrix* of F_i with respect to X_{i-1}.
- F_i has two Jacobian matrices, because it has to arguments.
- Element (k, l) of this Jacobian indicates how much the k-th output wiggles when we wiggle the l-th input.
- **The equation above is a recurrence equation!**

Jacobians and Dimensions

- derivatives with respect to a column vector are line vectors (dimensions: $[1 \times N_{i-1}] = [1 \times N_i] * [N_i \times N_{i-1}]$)

$$\frac{\partial E}{\partial X_{i-1}} = \frac{\partial E}{\partial X_i} \frac{\partial F_i(X_{i-1}, W_i)}{\partial X_{i-1}}$$

- (dimensions: $[1 \times N_{wi}] = [1 \times N_i] * [N_i \times N_{wi}]$):

$$\frac{\partial E}{\partial W_i} = \frac{\partial E}{\partial X_i} \frac{\partial F_i(X_{i-1}, W_i)}{\partial W}$$

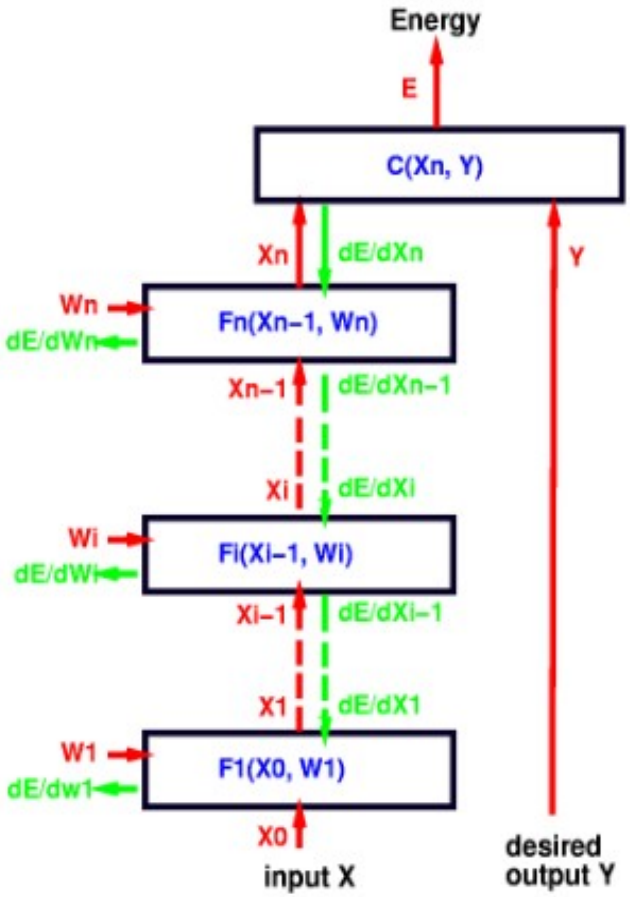
- we may prefer to write those equation with column vectors:

$$\frac{\partial E}{\partial X_{i-1}}' = \frac{\partial F_i(X_{i-1}, W_i)}{\partial X_{i-1}}' \frac{\partial E}{\partial X_i}'$$

$$\frac{\partial E}{\partial W_i}' = \frac{\partial F_i(X_{i-1}, W_i)}{\partial W}' \frac{\partial E}{\partial X_i}'$$

Back Propgation

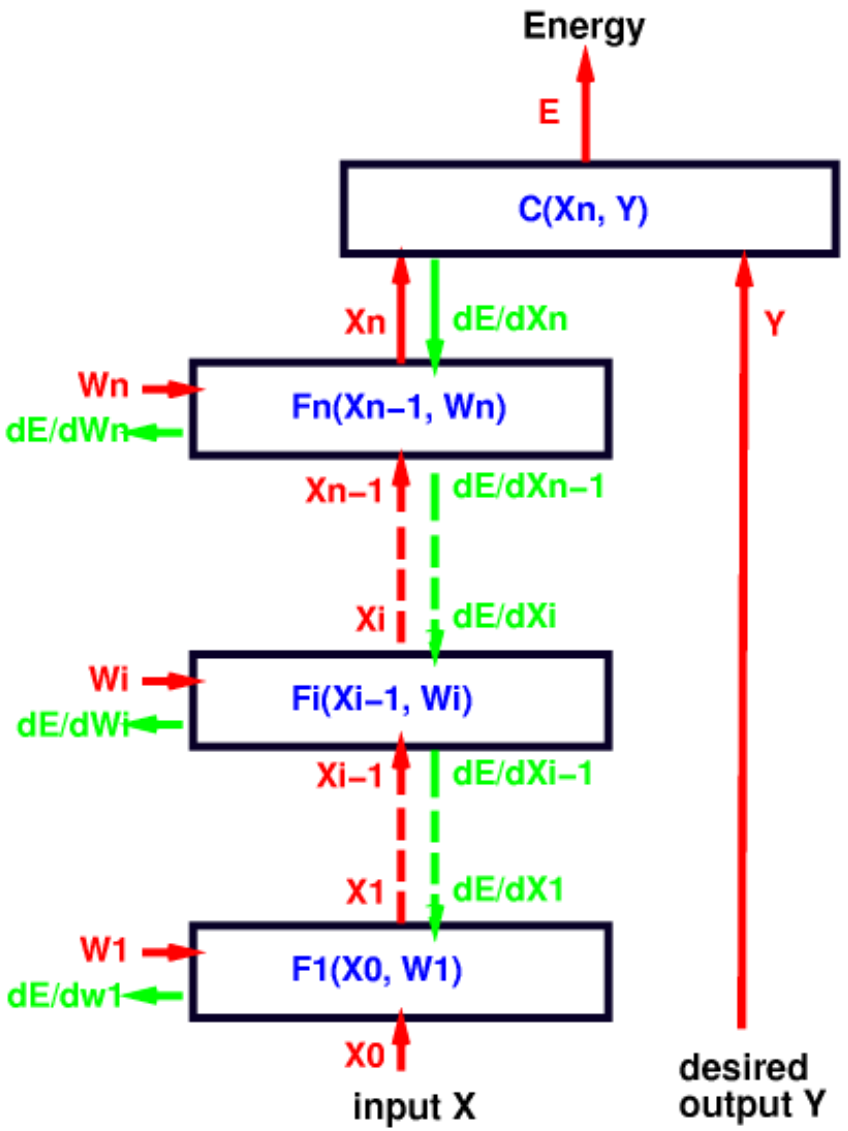
To compute all the derivatives, we use a backward sweep called the **back-propagation algorithm** that uses the recurrence equation for $\frac{\partial E}{\partial X_i}$



- $\frac{\partial E}{\partial X_n} = \frac{\partial C(X_n, Y)}{\partial X_n}$
- $\frac{\partial E}{\partial X_{n-1}} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(X_{n-1}, W_n)}{\partial X_{n-1}}$
- $\frac{\partial E}{\partial W_n} = \frac{\partial E}{\partial X_n} \frac{\partial F_n(X_{n-1}, W_n)}{\partial W_n}$
- $\frac{\partial E}{\partial X_{n-2}} = \frac{\partial E}{\partial X_{n-1}} \frac{\partial F_{n-1}(X_{n-2}, W_{n-1})}{\partial X_{n-2}}$
- $\frac{\partial E}{\partial W_{n-1}} = \frac{\partial E}{\partial X_{n-1}} \frac{\partial F_{n-1}(X_{n-2}, W_{n-1})}{\partial W_{n-1}}$
- ....etc, until we reach the first module.
- we now have all the $\frac{\partial E}{\partial W_i}$ for $i \in [1, n]$.

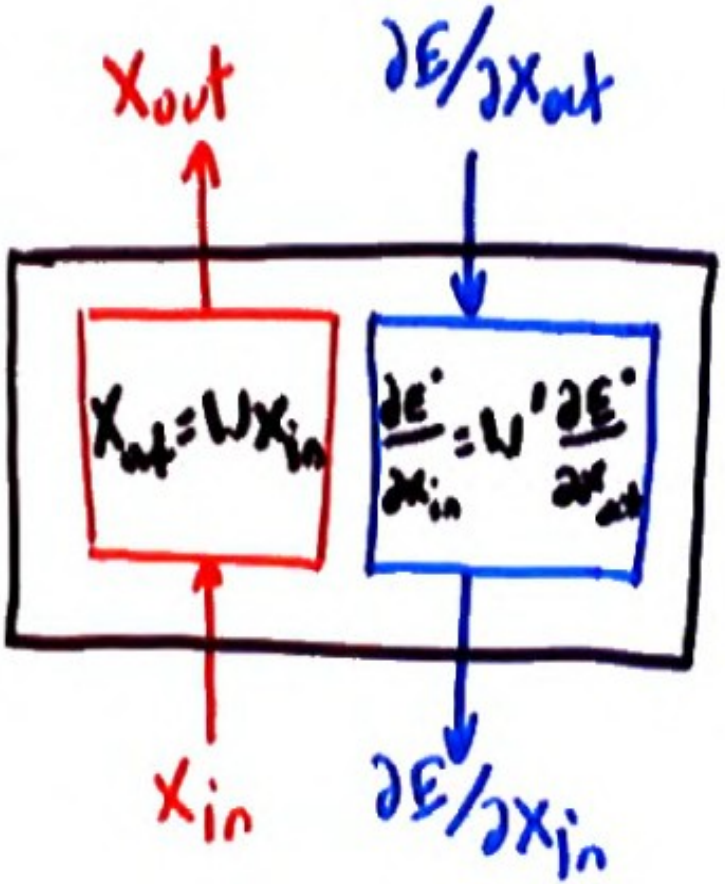
Multimodule Systems: Implementation

Y LeCun
MA Ranzato



- **Backpropagation through a module**
  - Contains trainable parameters
  - Inputs are arguments
  - Gradient with respect to input is returned.
  - Arguments are input and gradient with respect to output
- **Torch7 (by hand)**
  - `hidg = m2:backward(hid,outg)`
  - `ing = m1:backward(in,hidg)`
- **Torch7 (using the nn.Sequential class)**
  - `ing = model:backward(in,outg)`

The input vector is multiplied by the weight matrix.



- fprop: $X_{\text{out}} = W X_{\text{in}}$
- bprop to input:
$$\frac{\partial E}{\partial X_{\text{in}}} = \frac{\partial E}{\partial X_{\text{out}}} \frac{\partial X_{\text{out}}}{\partial X_{\text{in}}} = \frac{\partial E}{\partial X_{\text{out}}} W$$
- by transposing, we get column vectors:
$$\frac{\partial E}{\partial X_{\text{in}}}' = W' \frac{\partial E}{\partial X_{\text{out}}}'$$
- bprop to weights:
$$\frac{\partial E}{\partial W_{ij}} = \frac{\partial E}{\partial X_{\text{out}i}} \frac{\partial X_{\text{out}i}}{\partial W_{ij}} = X_{\text{in}j} \frac{\partial E}{\partial X_{\text{out}i}}$$
- We can write this as an outer-product:
$$\frac{\partial E}{\partial W}' = \frac{\partial E}{\partial X_{\text{out}}}' X_{in}'$$

Tanh module (or any other pointwise function)

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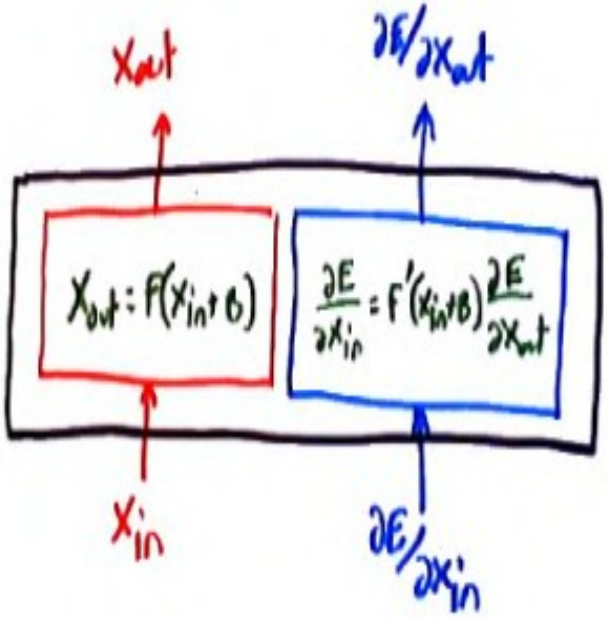
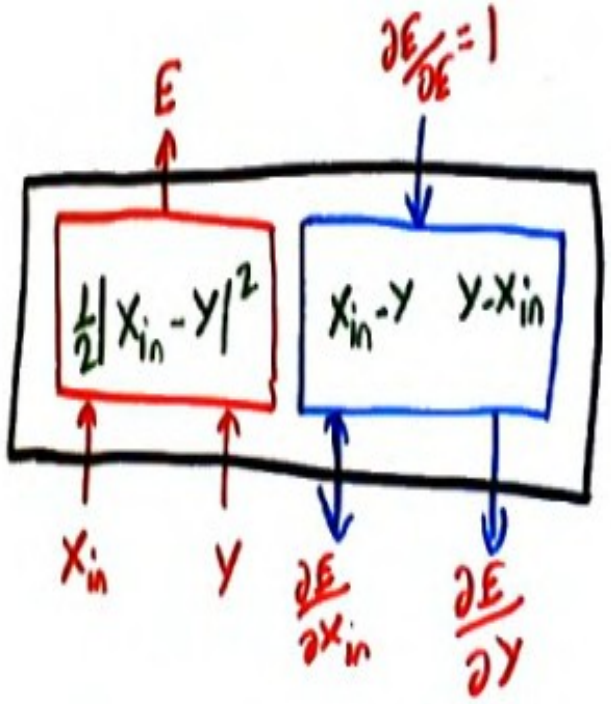


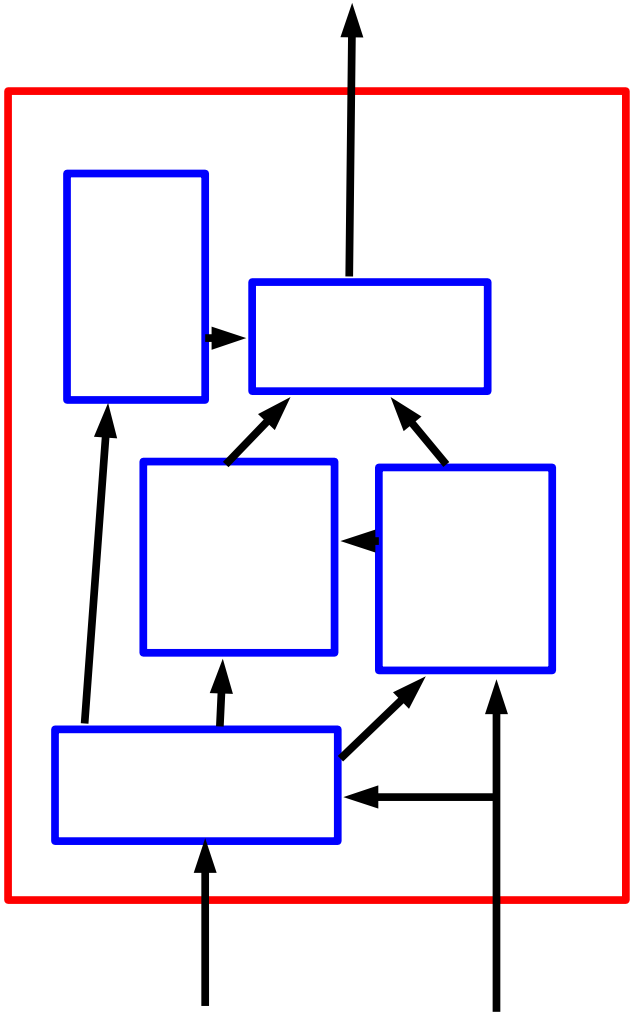
- fprop: $(X_{\text{out}})_i = \tanh((X_{\text{in}})_i + B_i)$
- bprop to input:
$$(\frac{\partial E}{\partial X_{\text{in}}})_i = (\frac{\partial E}{\partial X_{\text{out}}})_i \tanh'((X_{\text{in}})_i + B_i)$$
- bprop to bias:
$$\frac{\partial E}{\partial B_i} = (\frac{\partial E}{\partial X_{\text{out}}})_i \tanh'((X_{\text{in}})_i + B_i)$$
- $\tanh(x) = \frac{2}{1+\exp -x} - 1 = \frac{1-\exp(-x)}{1+\exp(-x)}$

Euclidean Distance Module



- fprop: $X_{\text{out}} = \frac{1}{2}||X_{\text{in}} - Y||^2$
- bprop to X input: $\frac{\partial E}{\partial X_{\text{in}}} = X_{\text{in}} - Y$
- bprop to Y input: $\frac{\partial E}{\partial Y} = Y - X_{\text{in}}$

Any Architecture works



- **Any connection is permissible**
  - Networks with loops must be "unfolded in time".
- **Any module is permissible**
  - As long as it is continuous and differentiable almost everywhere with respect to the parameters, and with respect to non-terminal inputs.

Module-Based Deep Learning with Torch7

Y LeCun
MA Ranzato

- **Torch7 is based on the Lua language**
  - Simple and lightweight scripting language, dominant in the game industry
  - Has a native just-in-time compiler (fast!)
  - Has a simple foreign function interface to call C/C++ functions from Lua
- **Torch7 is an extension of Lua with**
  - A multidimensional array engine with CUDA and OpenMP backends
  - A machine learning library that implements multilayer nets, convolutional nets, unsupervised pre-training, etc
  - Various libraries for data/image manipulation and computer vision
  - A quickly growing community of users
- **Single-line installation on Ubuntu and Mac OSX:**
  - curl -s https://raw.github.com/clementfarabet/torchinstall/master/install | bash
- **Torch7 Machine Learning Tutorial (neural net, convnet, sparse auto-encoder):**
  - http://code.cogbits.com/wiki/doku.php

Example: building a Neural Net in Torch7

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- Net for SVHN digit recognition
- 10 categories
- Input is 32x32 RGB (3 channels)
- 1500 hidden units
- Creating a 2-layer net
- Make a cascade module
- Reshape input to vector
- Add Linear module
- Add tanh module
- Add Linear Module
- Add log softmax layer
- Create loss function module

```
Noutputs = 10;
nfeats = 3; Width = 32; height = 32
ninputs = nfeats*width*height
nhiddens = 1500

-- Simple 2-layer neural network
model = nn.Sequential()
model:add(nn.Reshape(ninputs))
model:add(nn.Linear(ninputs,nhiddens))
model:add(nn.Tanh())
model:add(nn.Linear(nhiddens,noutputs))
model:add(nn.LogSoftMax())

criterion = nn.ClassNLLCriterion()
```

See Torch7 example at http://bit.ly/16tyLAx

Example: Training a Neural Net in Torch7

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```
for t = 1,trainData:size(),batchSize do
  inputs,outputs = getNextBatch()
  local feval = function(x)
    parameters:copy(x)
    gradParameters:zero()
    local f = 0
    for i = 1,#inputs do
      local output = model:forward(inputs[i])
      local err = criterion:forward(output,targets[i])
      f = f + err
      local df_do = criterion:backward(output,targets[i])
      model:backward(inputs[i], df_do)
    end
    gradParameters:div(#inputs)
    f = f/#inputs
    return f,gradParameters
  end   — of feval
  optim.sgd(feval,parameters,optimState)
end
```

- one epoch over training set
- Get next batch of samples
- Create a "closure" feval(x) that takes the parameter vector as argument and returns the loss and its gradient on the batch.
- Run model on batch
- backprop
- Normalize by size of batch
- Return loss and gradient
- call the stochastic gradient optimizer

Toy Code (Matlab): Neural Net Trainer

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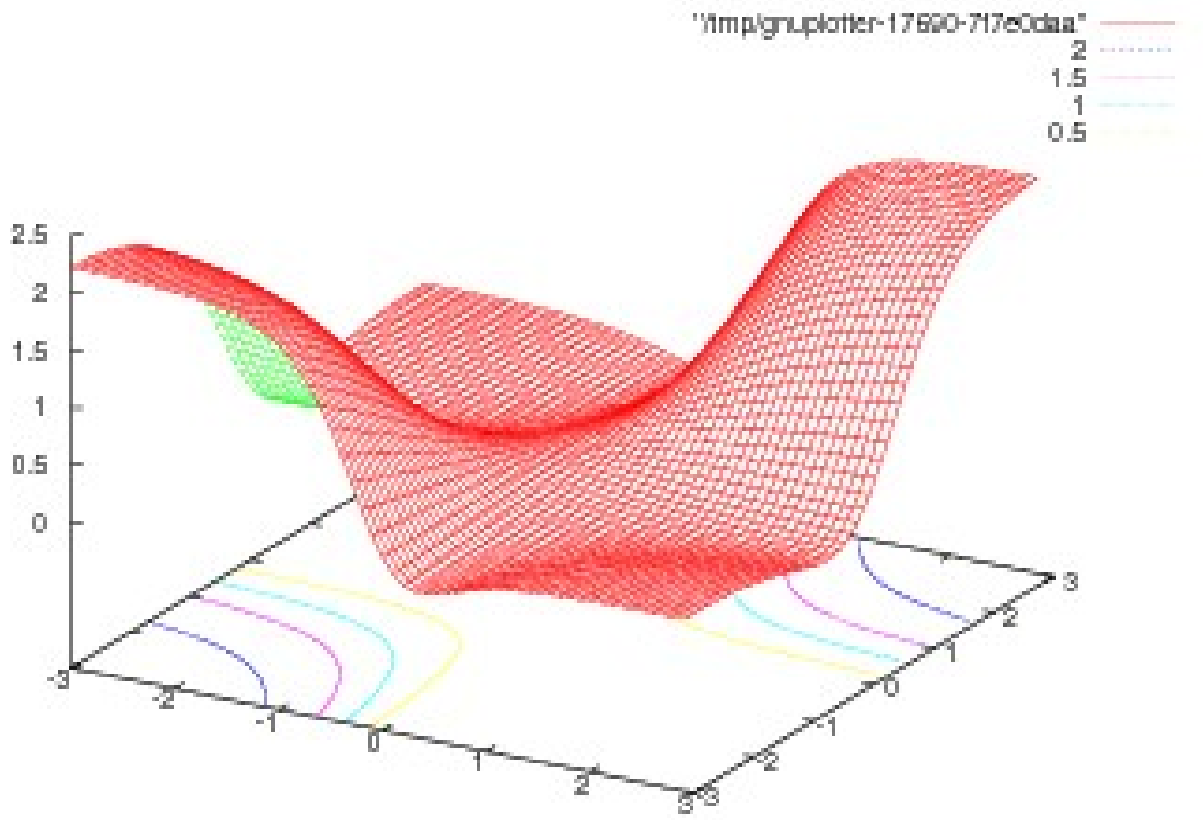
```
% F-PROP
for i = 1 : nr_layers - 1
  [h{i}  jac{i}]  =  nonlinearity(W{i} * h{i-1} +  b{i});
end
h{nr_layers-1}  =  W{nr_layers-1} * h{nr_layers-2}  +  b{nr_layers-1};
prediction  =  softmax(h{l-1});
% CROSS ENTROPY LOSS
loss  =  -  sum(sum(log(prediction)  .*  target)) / batch_size;
% B-PROP
dh{l-1}  =  prediction  -  target;
for i = nr_layers - 1 : -1 : 1
  Wgrad{i}  =  dh{i} * h{i-1}';
  bgrad{i}  =  sum(dh{i}, 2);
  dh{i-1}  =  (W{i}' * dh{i})  .*  jac{i-1};
end
% UPDATE
for i = 1 : nr_layers - 1
  W{i}  =  W{i}  -  (lr / batch_size)  *  Wgrad{i};
  b{i}  =  b{i}  -  (lr / batch_size)  *  bgrad{i};
end
```

Deep Supervised Learning is Non-Convex

Y LeCun
MA Ranzato

- **Example: what is the loss function for the simplest 2-layer neural net ever**
  - Function: 1-1-1 neural net. Map 0.5 to 0.5 and -0.5 to -0.5 (identity function) with quadratic cost:

$$y = \tanh(W_1 \tanh(W_0.x)) \quad L = (0.5 - \tanh(W_1 \tanh(W_0 0.5)^2$$



Backprop in Practice

- Use ReLU non-linearities (tanh and logistic are falling out of favor)
- Use cross-entropy loss for classification
- Use Stochastic Gradient Descent on minibatches
- Shuffle the training samples
- Normalize the input variables (zero mean, unit variance)
- Schedule to decrease the learning rate
- Use a bit of L1 or L2 regularization on the weights (or a combination)
  - But it's best to turn it on after a couple of epochs
- Use "dropout" for regularization
  - Hinton et al 2012 http://arxiv.org/abs/1207.0580
- Lots more in [LeCun et al. "Efficient Backprop" 1998]
- Lots, lots more in "Neural Networks, Tricks of the Trade" (2012 edition) edited by G. Montavon, G. B. Orr, and K-R Müller (Springer)

SOFTWARE

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MA Ranzato

Torch7: learning library that supports neural net training

– http://www.torch.ch
– http://code.cogbits.com/wiki/doku.php (tutorial with demos by C. Farabet)
- http://eblearn.sf.net (C++ Library with convnet support by P. Sermanet)

Python-based learning library (U. Montreal)

- http://deeplearning.net/software/theano/ (does automatic differentiation)

RNN

– www.fit.vutbr.cz/~imikolov/rnnlm (language modeling)
– http://sourceforge.net/apps/mediawiki/rnnl/index.php (LSTM)

CUDAMat & GNumpy

– code.google.com/p/cudamat
– www.cs.toronto.edu/~tijmen/gnumpy.html

Misc

– www.deeplearning.net//software_links

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– see yann.lecun.com/exdb/publis for references on many different kinds of convnets.

– see http://www.cmap.polytechnique.fr/scattering/ for scattering networks (similar to convnets but with less learning and stronger mathematical foundations)

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